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# The effect of disorder on symmetry breaking in quantum Hall systems

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## Abstract

We have been investigating quantum Hall systems by direct diagonalization and the density matrix renormalization group (DMRG). Calculations of the projected density–density correlation function for 10 electrons in 20 orbitals and 12 electrons in 24 orbitals indicate that disorder makes the correlation function behave more regularly with respect to system size. For these system sizes, randomness tends to enhance symmetry breaking when the wavelength of an additional imposed periodic potential is less than the wavelength of the stripe ordering. To investigate larger system sizes, preliminary calculations have been done using DMRG with momentum conservation in the  $y$ -direction. With our implementation of DMRG, promising results were obtained for the  $N = 0$  Landau level. It seems more difficult to obtain reliable results in the  $N = 2$  Landau level.

## 1. Introduction

The existence of anisotropic diagonal resistivity [1] in high ( $N \geq 2$ ) Landau levels is naturally attributed to the formation of stripes [2]. It is, however, quite challenging and interesting to do quantitative calculations of the behaviour (or even the existence) of these stripes and a variety of questions remain unsettled. In a previous paper [3] (hereafter referred to as DF) the  $N = 2$  Landau level at  $1/2$  filling was studied using direct diagonalization for systems of 10 electrons in 20 orbitals and 12 electrons in 24 orbitals for square clusters with periodic boundary conditions. The symmetry breaking, needed for stripe formation, was provided by a periodic potential in the  $x$ -direction, and consistent with mean field and weak coupling [4, 5] calculations, direct diagonalization indicated that stripes formed ‘perpendicular’ to the potential (the electron density going like  $\cos(k'y)$ ).

There were however, rather pronounced finite size effects. For example, the behaviour for a small periodic potential was quite different for the 10- and 12-electron cases. The 10-electron

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case suggested that an arbitrarily small periodic potential is sufficient to cause symmetry breaking, while the 12-electron system indicated that a finite strength of the potential was needed. Unfortunately, to overcome these finite size effects is difficult in that the number of system sizes accessible to direct diagonalization is limited. In an attempt to overcome these difficulties, in this paper, we will use a number of different approaches. First, in DF, a periodic potential  $A \cos(kx + \alpha)$  was used for a fixed phase  $\alpha$  and it was indicated that a limited amount of numerical experimentation showed little dependence on  $\alpha$ . In this paper, we will compute the equal time projected density–density correlation function  $S(q)$  for a fixed value of  $\alpha$  and then average over  $\alpha$ . For other problems of computational physics, averaging over a phase has had a beneficial effect in regard to finite size effects [6].

In any real experimental situation, a random potential is present. Therefore, we have investigated the effect of randomness and a periodic potential in 10- and 12-electron systems. If one supposes a large number of low-energy states, then the ground state is very sensitive to cluster size, since it is reasonable to assume the ordering of the low-lying states is determined in part by the cluster size. Randomness has a tendency to mix these states and therefore has the possibility of making finite size effects less severe. Finally, we have investigated larger system sizes, 14 electrons in 28 states, by direct diagonalization and a limited number of systems with up to 40 states by the density matrix renormalization group.

The paper is organized as follows. In the following section direct diagonalization results averaging over the phase of the potential and in the presence of an additional random potential are presented. The third section gives the details of our implementation of the density matrix renormalization group, comparisons to direct diagonalization and results for systems inaccessible to direct diagonalization. The final section is a summary and conclusions.

## 2. Numerical results for varying phase and disorder

Let us first investigate how averaging over the phase of the periodic potential effects stripe formation. The electrons interact with the long-range Coulomb interaction. We work in the  $N = 2$  Landau level at  $1/2$  filling for systems of 10 and 12 electrons. The notation of DF is used and we refer the reader to DF for further explanations. Throughout this paper, we work on a square cluster (as in DF) of sides ‘ $a$ ’ = ‘ $b$ ’ and allow potentials to break the  $x$ – $y$  symmetry. Whether periodic potentials actually are the source of symmetry breaking in ‘natural’ systems is a contentious issue [7, 8]; however, it may be possible to engineer Hall systems, either for semiconductors [9] or cold atoms [10], where periodic potentials are present. A general potential  $V(r)$  with Fourier transform  $V(q)$  adds a term to the Hamiltonian:

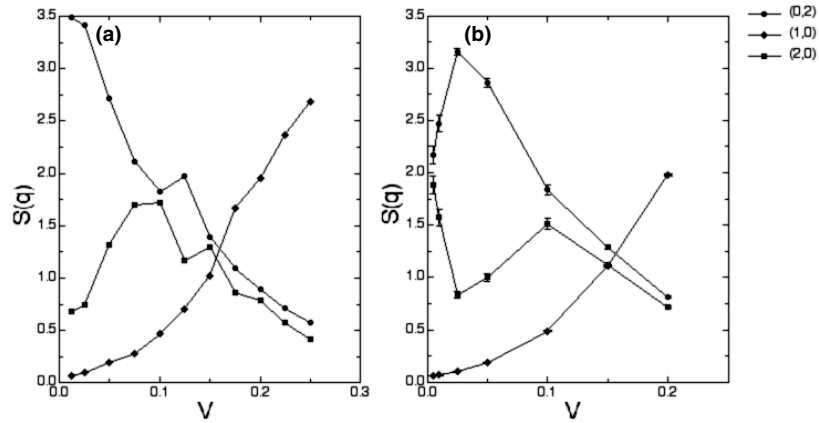
$$H_v = \sum_{l,k} f_{l,k} a_l^\dagger a_k$$

where

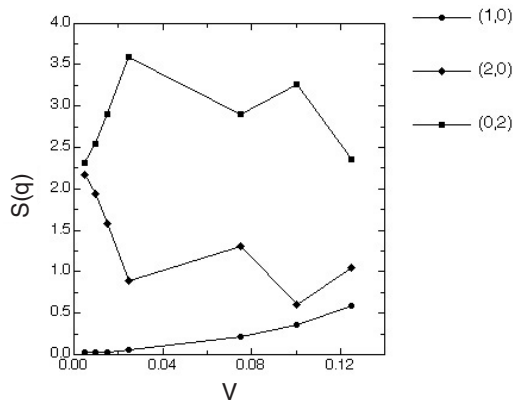
$$f_{l,k} = \frac{1}{ab} \sum_{\vec{q}} V(q) \exp\left(\frac{-q^2}{4}\right) L_2\left(\frac{q^2}{2}\right) \sum_M \delta(k - l + t - Mm) \exp\left(i\pi s \left(\frac{k}{m} + \frac{l}{m} + M\right)\right).$$

The choice made for the periodic potential is  $V(\vec{r}) = V(x) = 2V \cos\left(\frac{2\pi s}{a}x + \alpha\right)$  i.e.  $V(x)$  has one Fourier component of wavelength  $\lambda = \frac{a}{s}$  ( $q = \frac{2\pi s}{a}$ ) and hence  $f_{lk} = 2V \exp\left(-\frac{q^2}{4}\right) L_2\left(\frac{q^2}{2}\right) \cos\left(\alpha + \frac{2k}{m}\pi s\right) \delta_{lk}$ . This choice maintains momentum conservation in the  $y$ -direction and is convenient for numerical implementation. In DF,  $\alpha$  was chosen to be  $\alpha = \frac{-\pi s}{m}$ ; here an average over  $\alpha$  is taken.

To determine whether stripes exist and the direction of the stripes in the ground state wavefunction, we have computed the projected equal time density–density correlation function



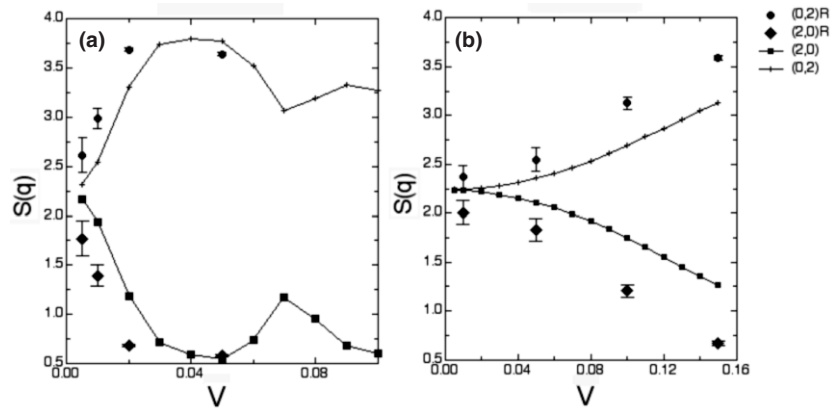
**Figure 1.**  $S(q)$  versus  $V$  for 10 electrons in 20 orbitals in the  $N = 2$  Landau level.  $V$  is the strength of the periodic potential with the period of the potential being  $a$ . In figure (a) the phase of the potential has been averaged over. In figure (b) a white noise potential has been added to the Hamiltonian; the error bars indicate the rms deviation. In both figures  $q = (\frac{2\pi n}{a}, \frac{2\pi m}{b})$  is abbreviated as  $(n, m)$ . The curves are a guide for the eye only; the symbols are the results of the calculation.



**Figure 2.**  $S(q)$  versus  $V$  for 12 electrons in 24 orbitals in the  $N = 3$  Landau level. The period of the potential is  $a$  and the phase of the potential has been averaged over.

$S(q, \alpha)$  for a fixed  $\alpha$  and then averaged over  $\alpha$ , that is  $S(q) = \frac{1}{2\pi} \int_0^{2\pi} d\alpha S(q, \alpha)$ . Figure 1(a) (2) shows  $S(q)$  for 10 (12) electrons in 20 (24) orbitals for a potential of wavelength  $\lambda = a$  as a function of the strength of the periodic potential  $V$ . Comparing to figures 2 and 3 in DF one notes only rather small quantitative differences between averaging and working at the fixed value of  $\alpha = \frac{-\pi s}{m}$ . In particular, for small values of  $V$  the behaviour of  $S(q)$  remains quite different for 10 and 12 electrons. In conclusion, averaging over the phase of the periodic potential makes little difference, and for the remainder of this paper  $\alpha$  is chosen to be the value  $\alpha = \frac{-\pi s}{m}$ . How then is stripe formation affected by randomness?

As in DF, the random potential was chosen to be delta correlated  $\langle U(r)U(r') \rangle = U_0\delta(r - r')$  and  $U_R \equiv \sqrt{\frac{3}{2}}U_0$ . In figure 1(b),  $S(q)$  is plotted versus the strength of the periodic potential for a periodic potential of wavelength  $\lambda = a$ ,  $U_R = 0.05$  and 10 electrons in 20 orbitals. Each point represents an average over at least 30 realizations of the random potential, the error bars showing the standard deviation. Since there is no momentum conservation, the computational effort is substantially greater. Comparing figure 1(b) to 1(a) (or figure 2 of DF),



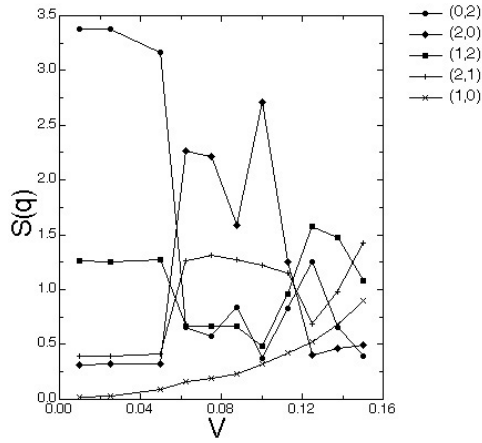
**Figure 3.**  $S(q)$  versus  $V$  for 12 electrons in 24 orbitals,  $N = 2$  Landau level. Figure (a) has a periodic potential of period  $a$ ; figure (b) is for a periodic potential of period  $a/5$ . The circles and diamonds are computed with a random potential; for the boxes and pluses no random potential is present.

one notes that randomness has a very pronounced effect on the behaviour for weak periodic potentials. The behaviour is more similar to 12 electrons in 24 orbitals without disorder, in that a threshold of potential strength has to be met before ‘good’ stripes are present. It therefore appears that this finite threshold behaviour is more ‘generic’, that the case of 10 electrons in 20 orbitals without disorder is in some sense unusual. To further examine this point, we have calculated  $S(q)$  for 12 electrons in 24 orbitals  $\lambda = a$  with disorder of strength  $U_R = 0.05$ . The results are presented in figure 3(a) along with a calculation for 12 electrons without disorder. We note that in the case of 12 electrons,  $S(q)$  appears to be stable to the effect of randomness. There is not a great difference between the absence or presence of the random potential. However, it appears that there is somewhat more symmetry breaking when the random potential is present. This is consistent with the results obtained by DF where randomness for one value of  $V$  appeared to enhance symmetry breaking for a periodic potential of period  $a/5$ . We have extended these results by examining the same  $U_R = 0.05$  with more averaging than DF (at least 30 realizations of the random potential) and for other values of  $V$ . In figure 3(b)  $S(q)$  is plotted with and without randomness. Qualitatively, the random potential does not appear to have a great effect (for small  $V$ ); however, quantitatively, the random potential appears to again enhance symmetry breaking. It is important to note that the wavevector of the stripes is still perpendicular to the wavevector of the periodic potential.

### 3. Larger system sizes

#### 3.1. 14 electrons in 28 orbitals by direct diagonalization

From the previous section, one sees that there is a reasonably coherent picture for 10 and 12 electrons. The crucial question remains: what happens for larger system sizes? Using direct diagonalization, it is possible to study 14 electrons in 28 orbitals without disorder. By using momentum conservation in the  $y$ -direction, this involves finding the ground state of matrices of rank  $\sim 1.5 \times 10^6$ , a feasible computational task on workstations. In figure 4,  $S(q)$  as a function of  $V$  is plotted for 14 electrons in 28 states. One observes the return of the ‘pathological’ behaviour of 10 electrons in 20 orbitals, namely, for arbitrary small  $V$  there is symmetry breaking. As we have argued previously, this behaviour is modified by disorder. To explicitly



**Figure 4.**  $S(q)$  versus  $V$  for 14 electrons in 28 orbitals in the  $N = 2$  Landau level. No random potential is added.

see this for 14 electrons is difficult since to compute one sample would take at least one week on a workstation, and averaging over 30 samples is needed. Secondly, for larger values of  $V$ ,  $S(q = (2, 0))$  takes on a fairly large value. This is consistent with the scenario predicted in [4] where for small  $V$  the stripes order perpendicular to the potential, while for larger  $V$  the ordering is parallel to the potential. Lastly, judging by the size of  $S(q = (0, 2))$  relative to other values of  $S(q)$ , symmetry breaking is weaker than for smaller systems. This is not consistent with what one would anticipate from simple arguments; namely, imagine one has a classical Wigner crystal, then for  $S(q) = 1/N \langle \sum_{i,j} e^{iq \cdot (r_i - r_j)} \rangle$   $q$  as a reciprocal lattice vector,  $i, j$  here specifies electrons but also are indices for the direct lattice. Therefore,  $e^{iq \cdot (r_i - r_j)} = 1$  and  $S(q) \sim N$ ; for classical stripes one would anticipate  $S(q) \sim N^{1/2}$ . What is the reason one does not see such behaviour? A possible model is that it is energetically favourable for the stripes to sit a fixed distance apart. Hence if the cluster size or shape is such that the stripes cannot sit at this distance, the stripes are destabilized.

In the 10-electron and 12-electron cases what the enhancement of  $S(q = (0, 2))$  is telling us is that two stripes are present in these system sizes. For 14 electrons, unfortunately, two stripes are too far apart to be very low in energy and three stripes are too close together to be energetically favourable. How large a cluster is needed to stabilize three stripes (for aspect ratio  $a/b = 1$ )? For 10 electrons the stripes are separated (centre to centre distance) by 5.6 (in units of the magnetic length  $l$ ). Hence to have three stripes one needs a cluster of length 16.8 and hence one needs to be able to treat 24 electrons in 48 orbitals. (Actually, according to these arguments, 45 orbitals are required; however, at  $1/2$  filling, it is better to treat 46 orbitals and we prefer to treat an even number of electrons). One possibility to treat such large clusters is the density matrix renormalization group [11] (DMRG). This method, the DMRG, was applied to Hall systems by Shibata and Yoshioka [12, 13]. In the next subsection, we present details of our implementation of the DMRG for Hall systems. We have provided these details because the method shows promise, but at least in our implementation, improvements would be desirable. By providing some details, it is hoped this will aid others in their efforts to make the DMRG for Hall systems a more useful tool.

### 3.2. Implementation of the DMRG

As in DF, we work in the Landau gauge with periodic boundary conditions in both the  $x$ - and  $y$ -directions. The single particle orbitals look like stripes, of width proportional to the magnetic length  $l$ , which are chosen to be oriented along the  $y$ -direction and are labelled 1 to  $n$ . This label

refers to both the position of the orbital in the  $x$ -direction and momentum in the  $y$ -direction. This choice of the gauge and boundary conditions makes it very natural to implement the DMRG, as the system appears to be one dimensional, albeit with long-range interactions. It is also natural, although one sees there are complications, to use conservation of momentum in the  $y$ -direction. Before one deals with the complications, what are the advantages? First, although we did not notice a great difference in performance as far as getting lower energies, when one explicitly conserves momentum there is a substantial speed-up. This is due to the fact that one can automatically set certain matrix elements to zero due to the conservation law and the sizes of various matrices are reduced. The second advantage is in the storage of arrays. When there is long-range interaction, for DMRG, it is advantageous to store arrays of the form  $\langle \lambda | c_{k_1}^+ c_{k_2}^+ c_{k_3} | \lambda' \rangle$  (more generally any three creation or annihilation operators). Here  $|\lambda\rangle$  and  $|\lambda'\rangle$  are block states and  $k_1, k_2, k_3$  are orbital indices. Since one knows the momentum of  $|\lambda\rangle, |\lambda'\rangle$ ,  $k_1$  and  $k_2$  then  $k_3$  is determined by momentum conservation. Hence rather than storing  $N^2 n^3$  numbers ( $N$  = number of states in the blocks) one needs only store  $N^2 n^2$  numbers. This is a substantial savings if the number of orbitals is 40.

We now proceed to a description of the infinite DMRG algorithm applied to Hall systems. One uses the infinite algorithm to grow the cluster size one desires to study, after which one does several ( $\sim 4$ ) finite system DMRG sweeps to obtain reasonably convergent results.

To aid in a pedagogical description of the infinite algorithm, let us say we want to grow a cluster with a concrete number of orbitals, say 36. One does the standard thing: start with a small chain of four orbitals. Immediately, complications arise: which four orbitals does one start with and which Hamiltonian does one use? The ambiguity is, that with periodic boundary conditions and long-range interactions, there are different Hamiltonians and different momenta depending on the cluster size. The ‘solution’ is to use the Hamiltonian for the final (36-site) cluster and the momenta for the final cluster but leaving out the orbitals in the middle [13]. Hence for the four-site cluster one chooses a chain with momenta 1 and 2 on the left and 35 and 36 on the right. One proceeds to grow the cluster in the natural way, adding states 3 and 34 in the second stage, 4 and 33 at the next stage, etc. The difficulty is that diagonalizing the Hamiltonian and finding the density matrix, etc, on a cluster with a hole in the middle is probably not such a good representation of states on a 36-orbital cluster. One might think that after sweeping several times of the finite algorithm what one inputs from the infinite algorithm does not matter much; however, we had found that the final answer is quite sensitive to the details of the infinite algorithm (see below). A partial cure for the ‘hole’ is to keep all states for the blocks of small clusters. The rationale here is that by the time one starts using the density matrix, etc, the cluster is large enough that the hole in the middle does not matter so much. In our implementation, we have been able to keep all the states for blocks with nine or fewer orbitals (i.e. 512 block states; it is probably feasible to extend this to 1024 states). Let us now see how the algorithm proceeds once one has reached the 512-state limit. We therefore start with the 20-site cluster with 10, 27 in the middle. The ground state of this cluster is computed using the Hamiltonian of the 36-orbital system (i.e. we emphasize that the 20-orbital Hamiltonian is not used). From the density matrix, the block states are determined in the standard way, as the states with largest density matrix eigenvalue. Typically, the number of block states selected (i.e. 160) is much less than 512. Different block states are used (and the relevant operators stored) for the right and left blocks; we find this gives a more stable result than ‘flipping’ say the left block states to get the right block states. This process is continued in the same way until the 36-orbital cluster has been ‘built up’. There are two possible ‘ambiguities’ in this process. First, in computing the ground state, it is meant to compute the ground state in each momentum sector and then select the state of lowest energy of all these states. Unfortunately, we found we could not fix the momentum of the ground state from the beginning of the calculation

**Table 1.** Comparison of DMRG to exact results in the  $N = 0$  Landau level. For 12 electrons in 36 orbitals no entry is given for the exact results since no exact results were available. For 16 electrons in 32 orbitals the exact results are due to Sheng [14].  $k$  denotes the momentum in the  $y$ -direction.

	# Electrons	# Orbitals	Energy	$\Delta E/E$	$k$
DMRG	10	30	-2.6841	$2.3 \times 10^{-3}$	5
Exact			-2.6902		5
DMRG	12	36	-3.3419		6
Exact					
DMRG	16	32	-5.2025	$4.7 \times 10^{-3}$	8
Exact			-5.2739		8

and get a very good (i.e. low-energy) result. The second trouble is that there can be too few nonzero eigenvalues of the density matrix to pick enough block states (i.e. we want to pick 160 block states but there are only, say, 100 nonzero eigenvalues). This is presumably due to the ‘hole’ in the middle of the cluster during the building-up stage. One possible cure is to add block states ‘by hand’. This is suggested by reference [13] but we have not been able to find a foolproof method for selecting the added block states, and the final results appear to be sensitive to what states are added. One possibility is to choose a ‘large’ system size, say 24 orbitals, calculate the density matrix and block states and add these states to smaller system sizes when there are too few nonzero eigenvalues. To be more precise, we have used the 24-orbital cluster with the hole in the middle (i.e. (1, 2, 3, ..., 12, 25, 26, 27, ..., 36) using the final Hamiltonian (i.e. the Hamiltonian for the 36-site cluster). Calculating the lowest energy state in a numerically exact way, for example using the Lanczos algorithm, one can then calculate the density matrix and the block states. Note that if there was ground state degeneracy, we picked one of the degenerate states. These block states can then be added to the infinite system DMRG calculation at ‘opportune’ times; for example, the 10-orbital block states are added to the 22-site DMRG cluster.

The above procedure seems to work when the final system size is fairly small, but does not seem to work consistently for larger final system sizes, i.e. 36 orbitals. In fact, we find that sometimes, just letting the computer select the 160 states with the highest eigenvalue (the lower states having eigenvalues  $0 + \text{noise}$ ) gives a better (lower-energy) result. Once the infinite algorithm is done, the finite algorithm proceeds in the standard way. Further details of the implementation of the finite algorithm are given below in the discussion of specific system sizes.

The preceding discussion probably gives the reader a too pessimistic view of the DMRG method applied to Hall systems. For a more ‘balanced’ view, in table 1, we present some DMRG results for the  $N = 0$  Landau level. In the DMRG calculation 160 states in a block have been used and the computation of 12 electrons in 36 states took roughly 4 days of cpu time using a 3 GHz processor (2 gigabytes of RAM and 20 gigabytes of hard disk were used, the relatively large amount of hard disk being necessary because of storage requirements due to the long-range interaction). The agreement with exact calculations for the energy is about 0.2% for 10 electrons in 30 orbitals and 1% for 16 electrons in 32 orbitals. Presumably, the better agreement at filling factor  $1/3$  is due to the gap between the ground state and the first excited state. The  $S(q)$  correlation functions (not shown) appear to be reasonable, i.e. liquid like, for 12 electrons in 36 orbitals.

Let us make some comments in regard to computational ‘details’ we used to obtain these results. First, in accord with DMRG intuition, the finite algorithm cycles are quite important. In the 12-electron, 36-orbital case, the energy decreased by about 10%,  $-2.93$  to  $-3.34$  and the momentum with the lowest energy changed from  $k = 5$  to 6 from the end of the infinite



algorithm to the final cycle of the finite algorithm. It is also important to note, for this system size, that during the finite algorithm the momentum  $k$  of the lowest energy state ranged from 5 to 11 before converging to 6. From our experience, two cycles of the finite algorithm seem to be essential and we have done four in total. In doing the infinite algorithm calculation, we computed the lowest energy state in each momentum sector and used the lowest energy state of all these states to compute the density matrix, etc. For the first two finite algorithm sweeps, we found we could take the  $k$  from the previous iteration and then consider all  $k$ s in the range  $(k - 4, k + 4)$ . For the last two sweeps, it was sufficient to fix the value of  $k$ . This permits a substantial speed-up of the calculation and makes no difference in the final result as the value of  $k$  does not change between iterations.

As noted earlier, the final energy is sensitive to the details of the infinite system algorithm, for example, how states are added. If we take the block states from a calculation of 8 electrons in 24 orbitals, using the 36-orbital Hamiltonian, and add states to systems with 20, 22 and 24 orbitals, we obtain an energy of  $-3.34189$  and  $k = 6$ . Note that even after the infinite algorithm reaches 26 orbitals, there is still a problem of small eigenvalues, i.e. adding states as we have done does not cure the problem. By adding states to a system with 20 orbitals, we mean, take the 20-orbital system, calculate the density matrix, keep all states with eigenvalue  $> 10^{-5}$  (this is the choice we made but it is unclear whether it is really optimal) and then make a set of 160 states adding the rest from the 8, 24 calculation, treating 10 orbitals as the system, 14 orbitals as the environment. On the other hand, if we add states from a 10-electron, 30-orbital calculation (again adding states at stage 20, 22, 24 in the finite system procedure) we obtain  $k = 18$  and an energy of  $-3.3326$ . The different momentum one gets is not particularly disturbing as one expects a three-fold degenerate ground state with  $k = 6, 18$  and 30. The difference in the energy of the two calculations is an indication of the error caused by keeping a limited number of states in the blocks. A stranger phenomenon is that the result is sensitive to how very small eigenvalues of the density matrix are treated. For example, if we make a random choice of the states with eigenvalue  $< 10^{-16}$ , after the infinite algorithm reaches a 26-orbital cluster, one obtains  $k = 18$  and an even higher energy  $-3.325$  for the ground state. Clearly, from these results, the DMRG method shows promise; however, a better way to deal with small eigenvalues of the density matrix is desirable.

Let us now consider smaller system sizes adding a periodic potential of wavelength  $\lambda = a$  and working in the  $N = 2$  Landau level. In table 2 the case of 10 electrons in 20 orbitals has been considered; here we did not ever keep any more than 160 states in the blocks, since if we kept 512 states the basis would not be at all reduced. States were added to the blocks if density matrix eigenvalues  $< 10^{-5}$  and the states were added from an exact calculation of 10 electrons in 20 states. Not surprisingly, the results from the DMRG agree rather well with exact calculations. Overall, however, quantitatively, the DMRG appeared to have more difficulty in the  $N = 2$  Landau level. As evidence of this, we have presented in table 3 a comparison of exact results to the DMRG for 14 electrons in 28 orbitals with 160 states in the blocks. Here the added states came from 12 electrons in 24 orbitals and up to 512 states were kept in the initial stage of the DMRG calculation. The energies obtained from the DMRG are not bad: there is agreement to better than 1%. However, the DMRG did not converge to a state with the correct momentum of the ground state. This might not be a fatal flaw since the agreement for the correlation function is generally good other than for  $V = 0.1$ .

To summarize, as opposed to the good (good in the sense that one converges to a state with the correct momentum) behaviour of the DMRG in the  $N = 0$  Landau level, the DMRG has problems in the  $N = 2$  Landau level. Bearing this in mind, we did a limited number of calculations for  $N = 2$ , 20 electrons in 40 orbitals, with 160 states in the blocks and  $V = 0.05$ . These calculations took about 10 days on a 3 GHz workstation. The lowest energy was obtained

**Table 2.** Comparison of DMRG to exact results in the  $N = 2$  Landau level for 10 electrons in 20 orbitals.  $(2, 0)$  denotes  $S(q = (2, 0))$ .

	Energy	$\Delta E/E$	(2, 0)	(0, 2)	$V$	$k$
DMRG	-1.501 03	$4.6 \times 10^{-4}$	0.67	3.50	0.01	5
Exact	-1.501 72		0.68	3.50		5
DMRG	-1.536 48	$3.8 \times 10^{-4}$	1.36	2.65	0.05	5
Exact	-1.537 06		1.34	2.69		5
DMRG	-1.638 64	$4.3 \times 10^{-5}$	1.84	1.71	0.10	1
Exact	-1.638 71		1.82	1.73		1
DMRG	-1.807 8713	$1.2 \times 10^{-6}$	1.30	1.39	0.15	13
Exact	-1.807 8735		1.30	1.39	0.15	13

**Table 3.** Comparison of DMRG to exact results in the  $N = 2$  Landau level for 14 electrons in 28 orbitals.

	Energy	$\Delta E/E$	(2, 0)	(0, 2)	$V$	$k$
DMRG	-2.4285	$1.0 \times 10^{-3}$	0.28	3.48	0.01	11
Exact	-2.4360		0.32	3.38		7
DMRG	-2.4636	$3.2 \times 10^{-3}$	0.27	3.18	0.05	5
Exact	-2.4716		0.32	3.17	0.05	19
DMRG	-2.5771	$3.4 \times 10^{-3}$	0.3	2.39	0.10	7
Exact	-2.5858		2.71	0.37		15

by not adding any additional states to the blocks. The energy of the state was  $-3.8626$  with the largest value of  $S(q) = S(\pm 2, \pm 2) = 2.86$ . Consistent with the exact 14-electron calculations in 28 states, the two-stripe state is unstable but the system is still not large enough for the three-stripe state to have a low energy. In fact, by adding states from a 12 electrons in 24 orbitals calculation to the blocks, the DMRG converged to a 20, 40 state that had an energy of  $-3.8102$  with the largest value of  $S(q) = S(3, 0) = 5.93$ ; that is, the DMRG converged to an excited state with a strong tendency to striped ordering.

What are some possibilities to improve the DMRG calculations? One idea is to perform the initial infinite system algorithm with a short-range potential to try to minimize the effect of the hole in the middle. Unfortunately, these calculations gave poorer results, higher energy, than using the long-range Coulomb potential. It was suggested to us some time ago, by Shibata [15], that it is quite important to keep a constant electron density during the course of the DMRG procedure. Of course, a constant electron density is the result of any exact calculation on a finite cluster with periodic boundary conditions (and no periodic potential). We, however, did not know how to implement this idea as the infinite system calculation broke the translational symmetry due to the hole in the middle (i.e. even in an exact calculation on the cluster  $(1 \dots 12, 25 \dots 36)$  the density is not constant). One simple possibility is to not use the actual wavefunction, but merely generate a ‘wavefunction’ in some momentum sector during each step of the infinite system algorithm using a random number generator for the coefficients. From this wavefunction, one can calculate the density matrix, the block states, etc, at every stage. The advantage of this approach is that a random wavefunction gives a number of large density matrix eigenvalues and the electron density is fairly constant. The energy of this wavefunction is rather high; however, a number of finite system sweeps can give a good energy. We tried this idea for  $N = 0$ , no periodic potential and 12 electrons in 36 orbitals for 160 states in the blocks. The energy we obtained was  $-3.359$  (picking the random wavefunctions to have  $k = 6$ ) with a momentum  $k = 30$ . This is to be compared to

our previous best of  $-3.342$  (see table 1). In addition, for 16 electrons in 32 orbitals the best energy obtained was  $-5.249$  with  $k = 8$ . It therefore seems quite promising to use random wavefunctions when building up the cluster. A possible ‘improvement’ would be to use the random wavefunction to ‘merely’ add block states to the block states already computed by using the actual Hamiltonian for the cluster; that is, not having to use block states from the actual cluster with very small eigenvalue of the density matrix. However, in our implementation, a purely random wavefunction gave lower energies.

#### 4. Conclusions

We have considered a few ways to come to grips with finite size effects in quantum Hall systems. The phase of an imposed periodic potential had a minor effect on the observables considered; however, a random potential seems to have the ability to partially tame finite size effects. A concrete result obtained in this regard is that randomness makes the density–density correlation function behave more regularly with respect to system size. This comes with a price, the price being that one no longer has momentum conservation, the state spaces are substantially larger and hence the numerical calculations are more difficult. Consistent with our earlier results, randomness tends to enhance symmetry breaking, in particular for the wavelength of the periodic potential (the case  $\lambda = a/5$ ) less than the wavelength of the stripes.

Ultimately, one needs to be able to treat larger sizes than are accessible by direct diagonalization. One of the most promising methods is the density matrix renormalization group (DMRG). Following Shibata and Yoshioka, the DMRG with momentum conservation in  $y$  has been implemented for quantum Hall systems. We were able to obtain promising results, in particular for the  $N = 0$  Landau level. It seems more difficult to obtain reliable results in higher ( $N = 2$ ) Landau levels.

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